VŠB – Technical University of Ostrava

Faculty of Mechanical Engineering

Department of Applied Mechanics

Dynamic Analysis of the Structure Using Substructure Decomposition

Dynamická analýza konstrukce technikou rozkladu na substruktury

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Diploma Thesis Assignment

Student: **Sinan Polat, BSc**

Study Programme: N2301 Mechanical Engineering

Study Branch: 3901T003 Applied Mechanics

Title: Dynamic Analysis of the Structure Using Substructure Decomposition
Dynamická analýza konstrukce technikou rozkladu na substruktury

The thesis language: English

Description:

1. Theoretical part - theory of substructure decomposition and modal analysis.
2. Design a model consisting of several parts that can be analyzed both individually and assembled.
3. FEM modelling and FEM modal analysis of the substructures.
4. Modal test of the substructures.
5. Comparison of computational and experimental models.
6. Creating the FEM model of the whole assembly using data of the substructures.
7. Modal test of the whole assembly.
8. Comparison of computational and experimental results.

References:

[1] O. C. Zienkiewicz. Computational Structural Analysis and Research Skills. McGraw-Hill, London, 1977. ISBN 0070840725; ISBN 9780070840720
[2] Thomson, W.T., Dahleh, M.D. : Theory of Vibration with Applications. Prentice Hall, Upper Saddle River, New Jersey, 1998. ISBN 0-13-651068-X

Extent and terms of a thesis are specified in directions for its elaboration that are opened to the public on the web sites of the faculty.

Supervisor: **doc. Ing. Jiří Podešva, Ph.D.**

Date of issue: 21.12.2018

Date of submission: 20.05.2019

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VYSOKÁ ŠKOLA BÁŇSKÁ TECHNICKÁ UNIVERZITA Fakulta strojní -5- OSTRAVA

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Dean

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Annotation of Diploma Thesis

POLAT, S. Dynamic Analysis of the Structure Using Substructure Decomposition: Diploma Thesis. Ostrava: VŠB –Technical University of Ostrava, Faculty of Mechanical Engineering, Department of Applied Mechanics, 2019, 46p. Thesis head: Podešva,J.

The dynamic analysis of the mechanical structures with large number of degrees of freedom (DOF) was in the past the problem of the machine time and disk space. On the today's hardware it is possible to solve 10^6 equations without big problem. But if the algorithm requires the iteration approach (non-linearity) or solving the dynamic problem via direct numerical integration this could take long time and large disk space. In this case the domain decomposition could give the advantage. The thesis describes the solution of the dynamic analysis of the structure with using so called "super-elements".

Key words: dynamic analysis, domain decomposition, sub-structuring.

Anotace

POLAT, S. Dynamická analýza struktury s využitím substrukturního rozkladu: Diplomová práce. Ostrava: Vysoká škola báňská - Technická univerzita Ostrava, Fakulta strojní, katedra aplikované mechaniky, 2019, 46p. Vedoucí práce: Podešva, J.

Dynamická analýza mechanických struktur s velkým počtem stupňů volnosti (DOF) byla v minulosti problémem času stroje a místa na disku. Na dnešním hardwaru je možné řešit 106 rovnic bez velkého problému. Pokud však algoritmus vyžaduje iterační přístup (nelinearitu) nebo řešení dynamického problému prostřednictvím přímé číselné integrace, může to trvat dlouho a velký prostor na disku. V tomto případě by mohlo být výhodou rozložení domény. Práce popisuje řešení dynamické analýzy struktury pomocí tzv. „Superprvků".

Klíčová slova: dynamická analýza, dekompozice domén, substrukturace.

Foremost, I would like to express my sincere gratitude to my advisor doc. Ing. Jiří Podešva, Ph.D. for the continuous support of my Master study, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Master study.

Besides my advisor, I would like to thank my teacher Mgr.Ing. Alena Bilošová, Ph.D. for her helps and mentoring during the experiments.

Last but not the least important, I owe more than thanks to my family members which includes my parents and brother, for their financial support and encouragement throughout my life

In addition, I am devoting all of my thesis work to my mother, who is inspiration of my studies.

Table of Contents

List of used signs and symbols

N	Number of substructures
M	Mass matrix
K	Stiffness matrix
f	External forces applied to the system in the time domain and in the frequency domain.
DOF	Degrees of freedom
FRF	Frequency Response Function
CMS	Craig-Bampton Method
FBS	Frequency Based sub-structuring
DS	Dynamic sub-structuring
$\tilde{K}$	Reduced stiffness matrix
$\tilde{f}$	Reduced force vector
$\tilde{M}$	Reduced mass matrix
$\tilde{B}$	Reduced damping matrix
q	Vector of unknown translations
SDOF	Single degree of freedom
MDOF	Multiple degree of freedom
Ω_0	Natural frequency[Hz]
$\{\Psi\}r$	Mode shape vector
k	Stiffness[Newton/meter]
m	Mass[kg]

1.INTRODUCTION

The dynamic analysis of mechanical parts is the study of the dynamic properties of the structures under vibrational excitation. It can be used to determine the vibrational characteristics of the part, such as natural frequencies, to evaluate the impact of the transient loads or to avoid noise and vibration problems with the design.

Performing this kind of analysis during the design stage can prevent or reduce the need and the costs of trials on test benches. Moreover, the failures under dynamic loads can be dramatic, so significant, costly errors, as well as the loss of brand reputation can be avoided.

Among the most common dynamic analyses, we can find:

- Modal analysis. This analysis is used to determine the natural frequencies of the part. It allows the engineer to develop the product with the certainty that the operational vibrations will never match the natural frequencies, with the intention to eliminate or, at least, minimize excessive vibrations that may result in critical failure.
- Harmonic response analysis. A follow on from modal analysis, harmonic analysis allows the evaluation of the part response to actual expected dynamic loads. E.g. stress, deflection and fatigue life of a part can be predicted based on dynamic loading.
- Transient dynamic analysis. In this analysis, the engineer can determine the response of the part due to loads that are a function of the time – similar to harmonic response except the loading can be non-periodic. It is commonly applied under conditions related to seismic or shock events.

In conclusion, static and dynamic finite element analysis can be used to "virtually prototype" designs before they are built, reducing risk, costs and speeding up time to market. [1]

The domain decomposition is the special approach to solving the large number of equations. It belongs to the group of elimination methods of strong decreasing the number of degrees of freedom (DOF).It consist in selecting the small number of DOF and solving this small system of equations. This takes short time and needs small disk space. The solution is then expanded to the original set of DOF. Dynamic Sub-structuring methods consist in dividing a system in subparts that can be analyzed separately then combing them together by an assembly procedure.

Such methods were first introduced four decades ago in order to reduce the complexity of dynamical models and to reduce the size of computational models. [2]

The dynamic sub-structuring methods have been used extensively in the past and many variants have been proposed over the years.

Although computer power has tremendously improved over the year allowing solving large problems and handling complex models, sub-structuring techniques are still very popular in engineering since they allow spreading the development work amongst different subgroups. Also, the models become more and more complex (in terms of number of degrees of freedom and in terms of physics modeled) reduction techniques are still necessary for instance when optimizing designs. The concept of sub-structuring is strongly related to domain decomposition methods which have become the corner stone of efficient parallel computing. [3]

Different methods of Dynamic Sub-structuring exist. Two different classes of sub-structuring methods can be distinguished.

- Time-domain based methods,
- Frequency-domain based methods

For the time domain based methods, each subsystem is described by a generalized mass, damping and stiffness matrix. In particular when the generalized substructure matrices are build using local modal properties one calls them Component Mode Synthesis (CMS). The modal synthesis technique determines the dynamic behavior of a coupled system on the basis of a normal mode description of the uncoupled systems. The most well known CMS technique is the Craig-Bampton method. [4]

For the frequency domain based methods on the other hand, each subsystem is described in terms of Frequency Response Functions (FRF's) of the uncoupled systems. This class is named Frequency Based Sub-structuring (FBS).

Modal synthesis methods are easy to implement whenever mass and stiffness matrices of the substructures are known theoretically, e.g. by a finite element model. However they are difficult to apply when dealing with experimental data. If modal synthesis methods are applied on experimental data, an identification technique has to be used in order to be able to determine the mass, damping and stiffness matrices of the subsystems. When experimental data are considered, FBS has some advantages:

It uses the measured frequency response functions directly, which implies that errors introduced by modal analysis and the errors caused by high mode truncation are eliminated.

Because the measured data represents the actual physical behavior of the structure, dependency of the structural dynamics on frequency (such as for visco-elastic materials) are included in the FRF measurements whereas they cannot be described by classical model synthesis approaches.

Experimental sub-structuring based on Frequency Based Sub-structuring approaches have become an important research issue in the last years [5]. The advantages of experimental sub-structuring are numerous:

It gives the possibility to combine modeled parts from either theoretical or numerical analysis, and measured components derived from experimental tests. Combining experimental and theoretical models is also referred to as hybrid analysis.

The effect of changing the properties of a subsystem on the assembled system can be analyzed efficiently. Also by analyzing the subsystems, local dynamic behavior can be recognized more easily than when the entire system is analyzed.

- It allows sharing and combining of substructures from different project groups.

When a substructure is changed, dynamic sub-structuring allows rapid evaluation of the dynamics of the complete system. Only the changed subpart needs to be measured and thereby allows efficient local optimization, fast design cycles and subsequently an overall optimization.

Dynamic Sub-structuring can be convenient if a measurement cannot be done because the structure is too large or complex to be measured as a whole or if not enough excitation energy can be put in the structure for adequate excitation.

- It allows easier spotting of local problems that might not be visible by testing the entire structure.

Dynamic Sub-structuring also has some disadvantages. The main disadvantages are:

- Applicability of Dynamic Sub-structuring is usually limited to linear and stationary systems with constant parameters.

For experimental sub-structuring, most measurements are limited to translational degrees of freedom because rotational degrees of freedom are difficult to measure. Assembling rotational dofs is thus a major challenge. [6]

- Dynamic sub-structuring code can take substantial time to program.

For experimental sub-structuring, measurements containing noise are used. The matrix inversion(s) that are needed in the algorithm(s) will propagate measurement noise, resulting in an inaccurate solution for the complete system.

Tackling these issues is essential if FBS techniques [7] are to become the methods of choice for efficient experimental analysis of structures in the future. One important drawback in improving the FBS method is the unnecessary complexity involved in all publications relative to the subject. Indeed, using proper dual formulations the mechanical interpretation of FBS as well as the mathematical formulation can be greatly simplified, opening new opportunities for future breakthroughs in experimental sub-structuring.

2.DOMAIN DECOMPOSITION

Dynamic sub-structuring (DS) has played a significant role in the field of structural dynamics and continues to be of great value. Performing the analysis of a structural system component wise has some important advantages over global methods where the entire

problem is handled at once:

1) It allows evaluating the dynamic behavior of structures that are too large or complex to be analyzed as a whole. For experimental analysis, this is true for large and complex systems such as aircraft. For numerical models, this holds when the number of degrees of

freedom is such that solution techniques cannot find results in a reasonable time.

2) By analyzing the subsystems, local dynamic behavior can be recognized more easily than when the entire system is analyzed. Thereby, DS allows identification of local problems as well as efficient local optimization. Also, dynamic sub-structuring allows the elimination of local subsystem behavior which has no significant impact on the assembled system. This results in a simple representation of the component's dynamics (e.g., an effective mass criteria) and, consequently, in an additional reduction of analysis time.

3) Dynamic sub-structuring gives the possibility of combining modeled parts (discretized or analytical) and experimentally identifying components.

4) It allows sharing and combining substructures from different project groups.

The goal of this paper is to present a general framework which allows for classification of dynamic sub-structuring methods and highlights the interrelations and differences between the many variants published. It is indeed peculiar that, despite the fact that dynamic sub-structuring concepts have been used and investigated for many years, such general overviews on the subject have only rarely been proposed. [8]

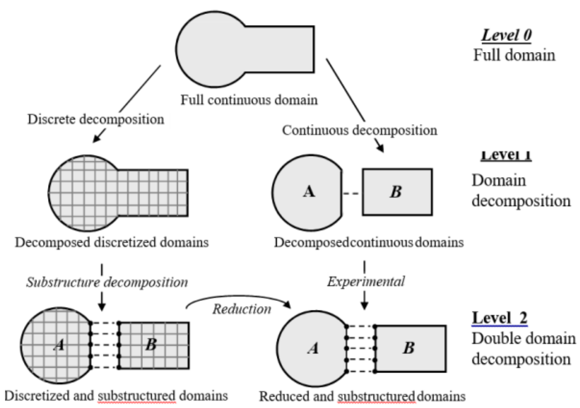

Figure 1- Dynamic sub-structuring and its relation to domain decomposition. [3]

The domain decomposition technique allows strongly decrease the number of DOF. The methods of reduction can be split into two groups.

The elimination method consist in eliminating (neglecting) the large number of DOF. The typical representative is the static condensation method.

The transformation methods consist in defining the totally new set of unknown coordinates (usually of no physical meaning) using transformation matrix. The typical representative is the modal transformation method.

The domain decomposition method belongs to the first group.

Consider the classic task of the linear static, written in matrix form.

$$\boldsymbol{K}.\boldsymbol{q} = \boldsymbol{f} \qquad (1)$$

Where K is the stiffness matrix, q is the vector of unknown translations and $\boldsymbol{f}$ is the vector of loading forces. Let us split the original set of DOF q into the subset q_m of so called ‘’master’’ DOF, which will be retained after reduction, and the sub-set q_s of so called ‘’slave’’ DOF, which will be eliminated. The mathematical record will then be:

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \cdot \begin{Bmatrix} q_m \\ q_s \end{Bmatrix} = \begin{Bmatrix} f_m \\ f_m \end{Bmatrix} \qquad (2)$$

or

$$K_{mm} \cdot q_m + K_{ms} \cdot q_s = f_m \qquad (3)$$

$$K_{sm} \cdot q_m + K_{ss} \cdot q_s = f_s \qquad (4)$$

If we will derive from the second group of equations:

$$q_s = K_{ss}^{-1} \cdot (f_s - K_{sm} \cdot q_m) \qquad (5)$$

After substitution:

$$\widetilde{K} = K_{mm} - K_{ms} \cdot K_{ss}^{-1} \cdot K_{sm} \qquad (6)$$

$$\tilde{f} = f_m - K_{ms} \cdot K_{ss}^{-1} \cdot f_s \qquad (7)$$

The equations have the same form as the original equations.

$$\widetilde{K} \cdot q_m = \tilde{f} \qquad (8)$$

Here $\widetilde{K}$ is the reduced stiffness matrix and $\tilde{f}$ is the reduced force vector.The solution can be extended by the reduced mass matrix $\widetilde{M}$ and reduced damping matrix $\tilde{B}$ into the area of linear dynamics. [9]

To the above written we must note that while the original stiffness matrix K is narrow strip and sparse, the reduced stiffness matrix $\widetilde{K}$ is full. That is why the set of master DOF must be as small as possible.

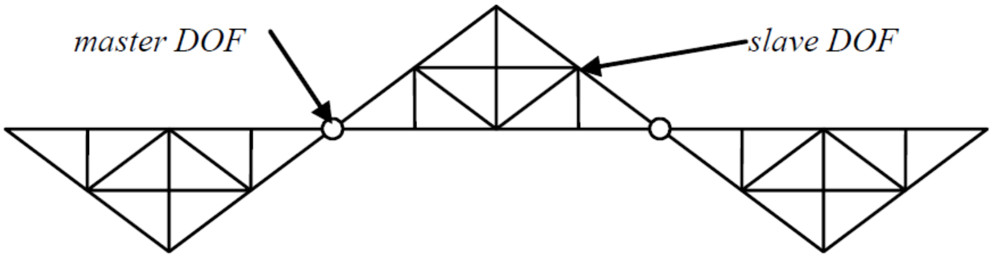

Figure 2- The main structure divided into three sub-structures [18]

If the mechanical structure can be naturally dividing into a few sub-structures, these will be the sub-domains. The sub-structures are joined together in the narrow boundaries of the very small number of DOF. These interface DOF will be retained as masters, interior DOF will be hidden as slaves.

The reduced stiffness matrix $\widetilde{K}$ of such structure represents the stiffness matrix of the structure in which the single sub-structure seems to be single finite element. However because in real they are rather large-scale systems they are called''super-elements''

The sub-domains (super-elements) must be internally linear. If the super-elements are used to build the ‘’macro model’’, this can contain also elements of the other types, including contact elements and other non-linearities.

For dynamic analysis we also need the reduced mass matrix. It can be expressed as

$$\widetilde{M} = M_{mm} - M_{ms} \cdot K_{ss}^{-1} \cdot K_{sm} - K_{ms} \cdot K_{ss}^{-1} \cdot M_{sm} + K_{ms} \cdot K_{ss}^{-1} \cdot M_{ss} \cdot K_{ss}^{-1} \cdot K_{sm} \ (9)$$

The equations of motion are then :

$$\widetilde{M} \cdot \ddot{q_m} + \widetilde{K} \cdot q_m = \tilde{f} \qquad (10)$$

In this case not only the boundary nodes will be selected as masters (see fig. 2) but also nodes inside the substructures.

2.1.General Framework for Dynamic Sub-structuring

This section is focusing on sub-structuring methods in general and not only in the context of model reduction. In the framework proposed here, the structural dynamics are therefore analyzed in three distinct domains: the physical, modal, and frequency domains.

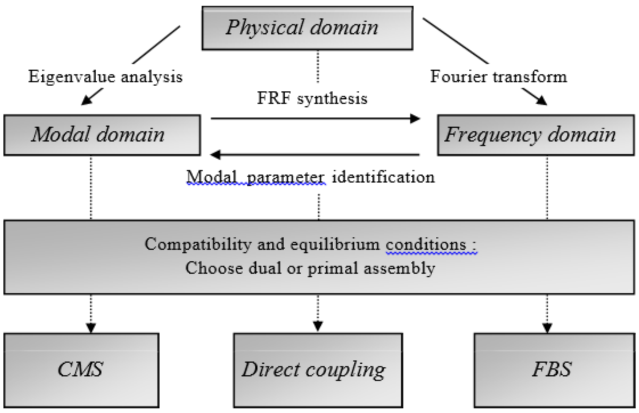

Figure 3- Representation of system dynamics in three domains [17]

In the physical domain, the structure is characterized by its mass, stiffness, and damping distributions, which are given by the corresponding stiffness, mass, and damping matrices for a discretized linear(ized) model. A structure in the frequency domain is seen through its frequency response functions. In the modal domain, the dynamic behavior of a structure is interpreted as a combination of modal responses: the system matrices are projected on the modal basis which, generally speaking, can be any basis representing the structural response. From a theoretical perspective, the same information is contained in all different representations(assuming no model reduction is performed). [10] This is schematically shown in Fig.3.

Substructures are structures that interact with their neighboring structures. When two or more substructures are to be coupled, two conditions must always be satisfied, regardless of the coupling method used:

1) Compatibility of the substructures' displacements at the interface is the so-called compatibility condition.

2) Force equilibrium on the substructures' interface degrees of freedom is called the equilibrium condition.

2.2.Sub-structuring With Using the Ansys Software

The Ansys software a complete substructure analysis involves three steps or passes:

- Generating the super-element(Generation Pass):This is where you condense a group of regular finite elements into a super-element. The condensation is done by identifying a set of master degrees of freedom. The master DOF are mainly used to define the interface between the super-element and other elements.
- Using the super-element (Use Pass): This is where you use the super-element in an analysis by making it part of the model. The model may contain just super-elements or a combination of super-elements and other ''regular'' elements (non-super-elements). The solution from the use pass consists of the reduced solution for the super-elements (i.e.,the degree of freedom solution at the master DOF) and the complete solution for any non-super-elements.
- Expanding results in the super-element (Expansion Pass): This is where you start with the reduced solution and back-calculate the results at all degrees of freedom in the super-element. If multiple super-elements were used in the use pass ,a separate expansion pass will be required for each super-element. [11]

3.BASIC THEORY

Modal analysis is the process of determining the modal parameters of a structure for all modes in the frequency range of interest. The ultimate goal is to use these parameters to construct a modal model of the response. Two observations worth noting here are that:

- Any forced dynamic deflection of a structure can be rep- resented as a weighted sum of its mode shapes.
- Each mode can be represented by an SDOF model.

3.1.Single-degree-of-freedom (SDOF) Models

As each peak - or mode - in a structural response can be represented by an SDOF model, we will look at some aspects of SDOF dynamics. In particular, we will examine the way in which SDOF structure can be modelled in the physical, time and frequency domains. These models are not intended to represent physical structures, but will serve as instruments for interpreting dynamic behavior (constrained by a set of assumptions and boundary conditions). [12] They will help us to:

- understand and interpret the behaviour of structures;
- describe the dynamic properties of structures, using a small set of parameters;
- extract the parameters from measured data (curve-fitting). An analytical model can be constructed in the physical domain. It is an abstract system consisting of a point mass (m). supported by a massless linear spring (k) and connected to a linear viscous damper (c). The mass is constrained so that it can move in only one direction (x) – a Single-degree-of-freedom. It can be seen on fig.4.

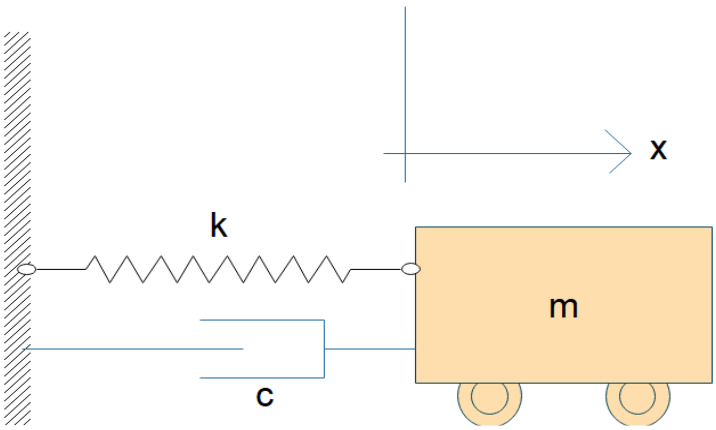

Figure 4- Single degree of freedom

•••• A mathematical model in the time domain can be derived by applying Newton's Second Law to the analytical model. By equating the internal forces (inertia, damping and elasticity) with the external (excitation) force, we obtain the model

$$m\ddot{x} + c\dot{x} + kx(t) = 0 \quad (11)$$

$$x(t) = C \cdot e^{-\delta t} \cdot sin(\Omega t + \varphi_0) \quad (12)$$

$$\Omega_0 = \sqrt{\frac{k}{m}} \quad (13) \qquad \delta = \frac{c}{2 \cdot m} \quad (14) \qquad \Omega = \sqrt{\Omega_0^2 - \delta^2} \quad (15)$$

which is a second-order differential equation. A model which is more mathematically manageable can be obtained in the frequency domain. [13]

3.2.The DOF and Multiple-degree-of-freedom (MDOF) Models

Real structures have many points which can move independently many degrees-of-freedom. To make an FRF measurement on a real structure we have to measure the excitation and response between two points. But any point may have up to six possible ways of moving so we must also specify the measurement direction.

•••• A degree-of-freedom (DOF) is a measurement-point-and- direction defined on a structure. An index i is used to indicate a response DOF, and j an excitation DOF. Additional indices x, y and z may be used to indicate the direction.

Thus $$\mathrm{Hij}(\omega) \equiv \frac{\mathrm{Xi}(\omega)}{\mathrm{Fj}(\omega)} \quad (16)$$

By writing Hij (ω) in two different ways, we obtain the two MDOF models shown as equations in the illustration. [14]

•••• The MDOF FRF-model represents Hij(ω) as the sum of SDOF FRFs, one for each mode within the frequency range of the measurement, where r is the mode number and m is the number of modes in the model.

•••• The MDOF modal-parameter model defines Hij(ω) in terms of the pole locations and residues of the individual modes. This model indicates two significant properties of the modal parameters:

- Modal frequency and damping are global properties.The pole location has only a mode number (r) and is independent of the DOFs used for the measurement.

- The residue is a local property. The index (ijr) relates it to a particular combination of DOFs and a particular mode.

In fig.5. two degrees of freedom can be seen.

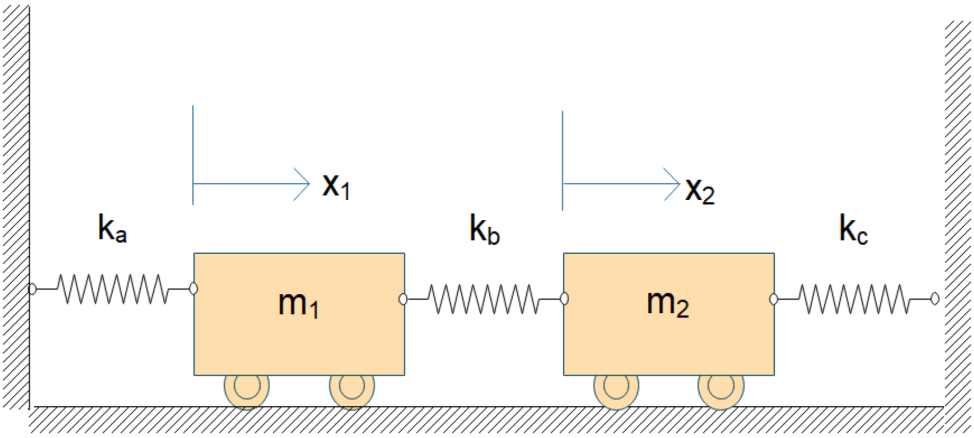

Figure 5- sketch of two degrees of freedom

In the figure, system consisting masses (m_1 and m_2) supported by massless linear springs (k_1,k_2 and k_3) and the directions shown by x_1 and x_2 .

For the mathematical model of two degrees of freedom we can consider this equations;

$$\boldsymbol{M} \cdot \ddot{x} + \boldsymbol{K} \cdot x = 0 \qquad (17)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (18)$$

3.3.What is a Mode Shape?

A mode shape is, a deflection-pattern associated with a particular modal frequency – or pole location. It is neither tangible nor easy to observe. It is an abstract mathematical parameter which defines a deflection pattern as if that mode existed in isolation from all others in the structure. [14] The actual physical displacement, at any point, will always be a combination of all the mode shapes of the structure. With harmonic excitation close to a modal frequency, 95% of the displacement may be due to that particular mode shape, but random excitation tends to produce an arbitrary "shuffling" of contributions from all the mode shapes. Nevertheless, a mode shape is an inherent dynamic property of a structure in "free" vibration (when no external forces are acting). It represents the relative displacements of all parts of the structure for that particular mode.

•••• Sampled mode shapes to the mode shape vector

Mode shapes are continuous functions which, in modal analysis, are sampled with a "spatial resolution" depending on the number of DOFs used. In general they are not measured directly, but determined from a set of FRF measurements made between the DOFs.A sampled mode shape is represented by the mode shape vector $\{\Psi\}_r$, where r is the mode number.

•••• Modal displacement

The elements Ψ_{ir} of the mode shape vector are the relative displacements of each DOF(i). They are usually complex numbers describing both the magnitude and phase of the displacement. [15]

4.MODEL

The model consists of 3 beams on top of each other and the beams are connected to each other by spring. During the experiment, the assembly was fixed to a plane with a clamp and kept still. The experimental measurements and the particulars of this measurements as frame body, used equipment, mesh and other will be described in this part.

4.1.Experimental Model

The model used in the experiment can be seen in the following figures (6,7,8);

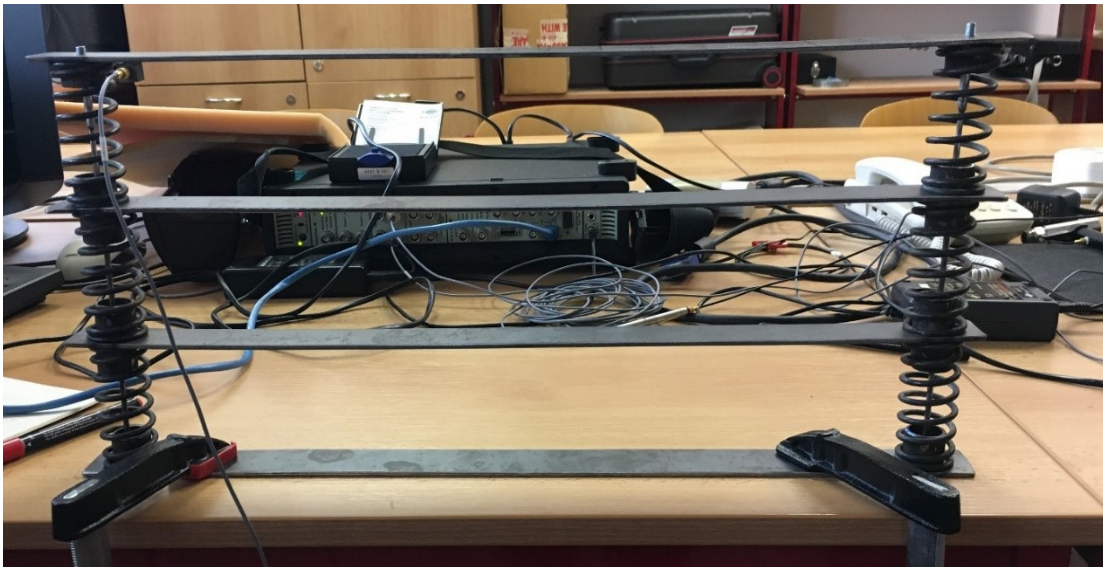

Figure 6-Front view of the model

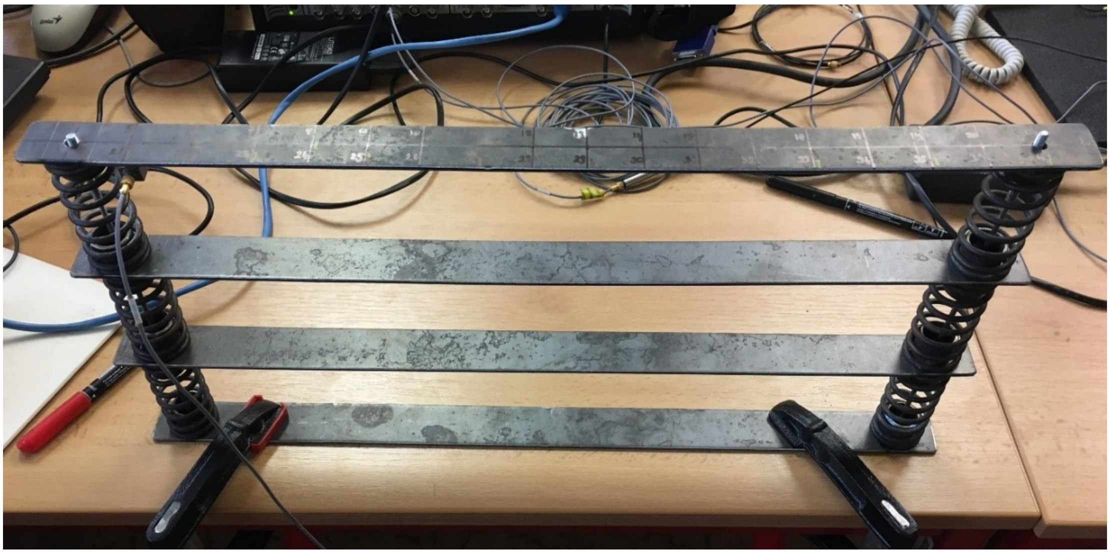

Figure 7-Isometric view of the model

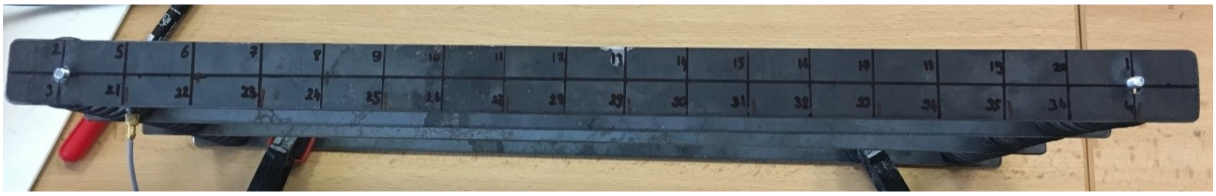

Figure 8-Top view of the model

The object of project is an assembly which is including 3 beams.The main dimensions of the measured frame are listed below.

4.2.FEM Model

Elements fall into four major categories: 2D line elements, 2D planar elements, and 3D solid elements which are all used to define geometry; and special elements used to apply boundary conditions. For example special elements might include gap elements to specify a gap between two pieces of geometry. Spring elements are used to apply a specific spring constant at a specified node or set of nodes. Rigid elements are used to define a rigid connection to or in a model. Most FEA tools support additional element types as well as somewhat different implementations of even these common elements.

- 1D Beam Element (line): Truss elements are long and slender, have 2 nodes, and can be oriented anywhere in 3D space. Truss elements transmit force axially only and are 3 DOF elements which allow translation only and not rotation. Trusses are normally used to model towers, bridges, and buildings. A constant cross section area is assumed and they are used for linear elastic structural analysis.
- 2D Shell Element (planar): 2D Elements are 3 or 4 node elements with only 2 DOF, Y and Z translation, and are normally created in the YZ plane. They are used for Plane Stress or Plane Strain analyses. Common applications include axisymmetric bodies of revolution such as missile radomes, radial seals, etc. and long sections with constant cross sectional area such as a dam. Plane Stress implies no stress normal to the cross section defined - strain is allowed - suitable to model the 2D cross section of a body of revolution. Plane Strain implies no strain normal to the cross section defined - stress is allowed - suitable to model the 2D cross section of a long dam.

- 3D Brick Element (body): Brick or tetrahedron elements may have 4, 5, 6, 7, 8, 15, or 20 nodes and support only translational DOF. They are normally used to model solid objects for which plate elements are not appropriate. You can usually specify either all tetrahedron, all bricks, or a mixture of both with some automatic mesh generators. This is the most common, and frequently the only element type supported by automatic mesh generators. Bricks work quite well for any "blocky" structures which are typical of machined, cast, or forged fabricated parts. Structural and thermal bricks exist so the same model geometry can be used for both the initial steady state heat transfer and subsequent thermal stress computations. Bricks compute stress through the thickness of a part. [16]

The FEM Models were created with using ANSYS MECHANICAL APDL on modelling section as Beam Element.

4.2.1.Single Beam

Single beam solution defined as free beam (no supports) can be seen on figure 9.

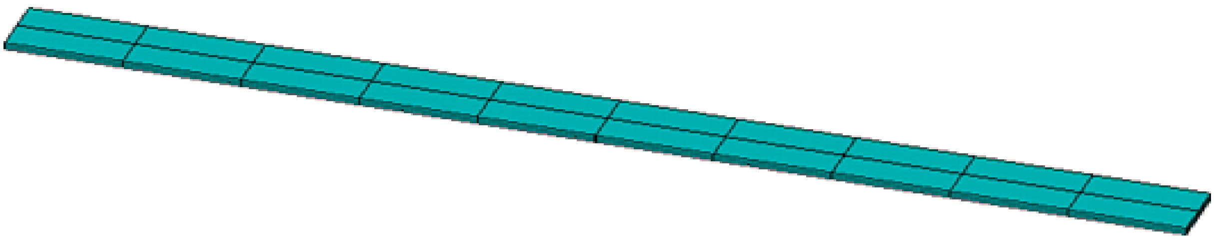

Figure 9- Single Beam

The real body was weight and the density of the material model was modified to reach the same weight as the real body. The natural frequencies and mode shape of the real beam was measured and the results was compared with these calculated. The Young modulus of the material model was modified to reach the same 1st natural frequency as was measured.

4.2.2.The Assembly

The assembly created with using ANSYS MECHANICAL APDL on modelling section. Single beams connected with spring (shown by purple lines in figure 10) and all red points belongs to one end of beams. These points reprensent supporting points. The directions can be seen in the figure 10 with red lines.

For points A,B and C assembly is fixed in X and Z direction also for these points rotation about X is fixed.

For points D,E and F assembly is fixed in Z direction.

Points G and H, which can be seen on the ground, they are fixed in X,Y and Z direction.

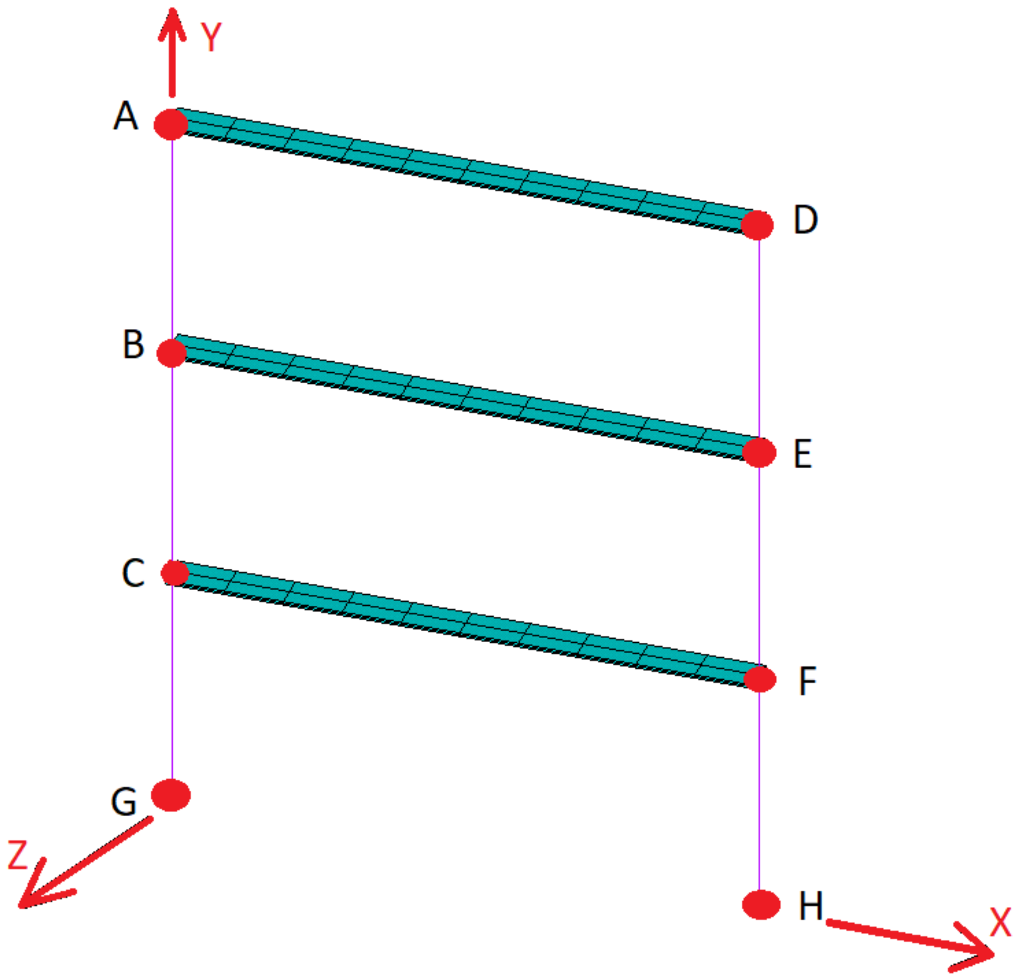

Figure 10-The model of assembly in ANSY APDL

5.MODAL ANALYSIS

5.1.FEM Modal Analysis for Single Beam

FEM Modal Analysis for single beam done by using ANSYS MECHANICAL APDL.First 3 bending are shown below on the figures.(11,12,13)

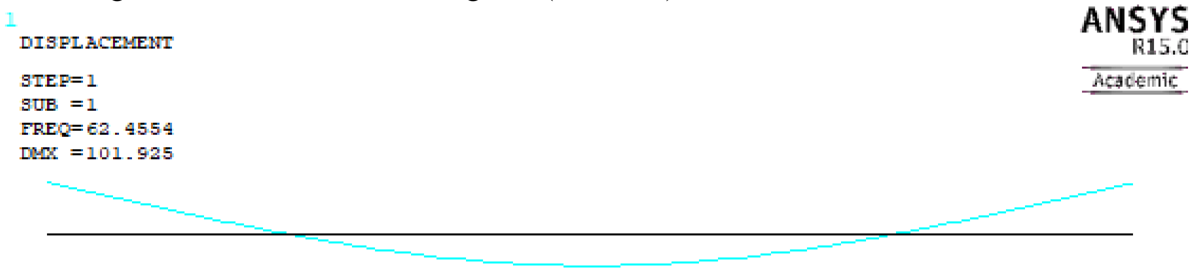

Figure11-1st bending for single beam

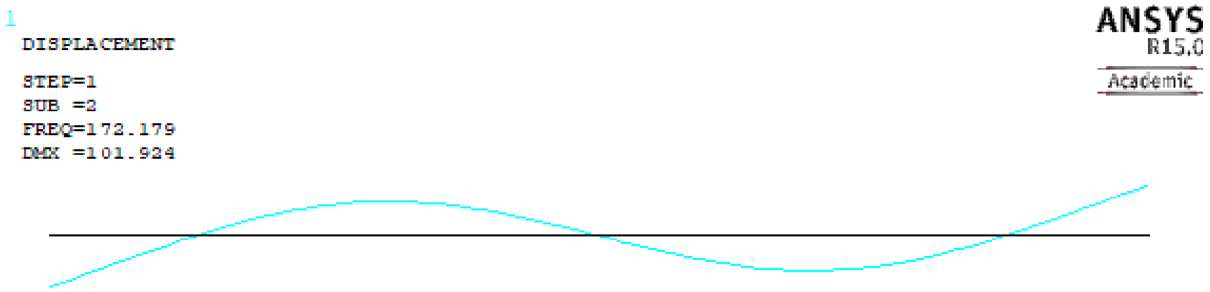

Figure 12-2nd bending for single beam

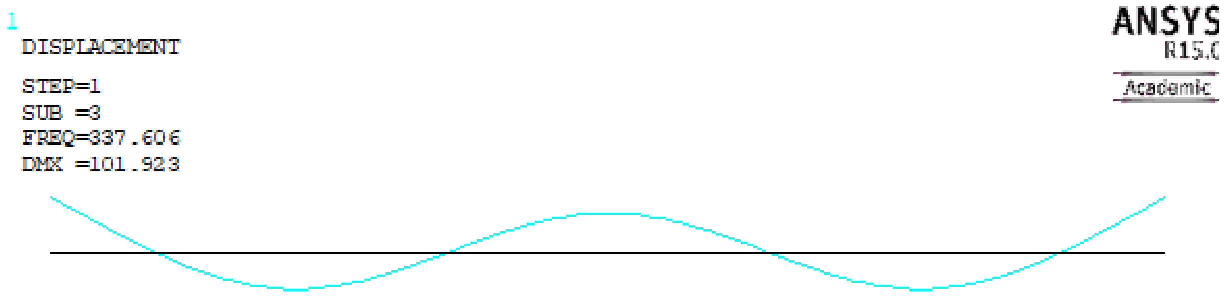

Figure 13-3rd bending for single beam

And all frequencies(Hz) are listed below on table 1.

Table 1-Frequencies for one beam

Mode	Frequency(Hz)	LOAD STEP
1	62.455	1. bending
2	172.18	2. bending
3	337.61	3. bending
4	558.24	4. bending
5	587.90	1. torsion
6	595.39	1. bending in horizontal plane
7	834.22	5. bending
8	1165.7	6. bending
9	1175.9	2. torsion

5.2.Experimental Modal Analysis for Single Beam

Experimental Modal Analysis was performed with using Brüel & Kjar-PULSE application. The geometry is firstly defined as shown in the figure 14.

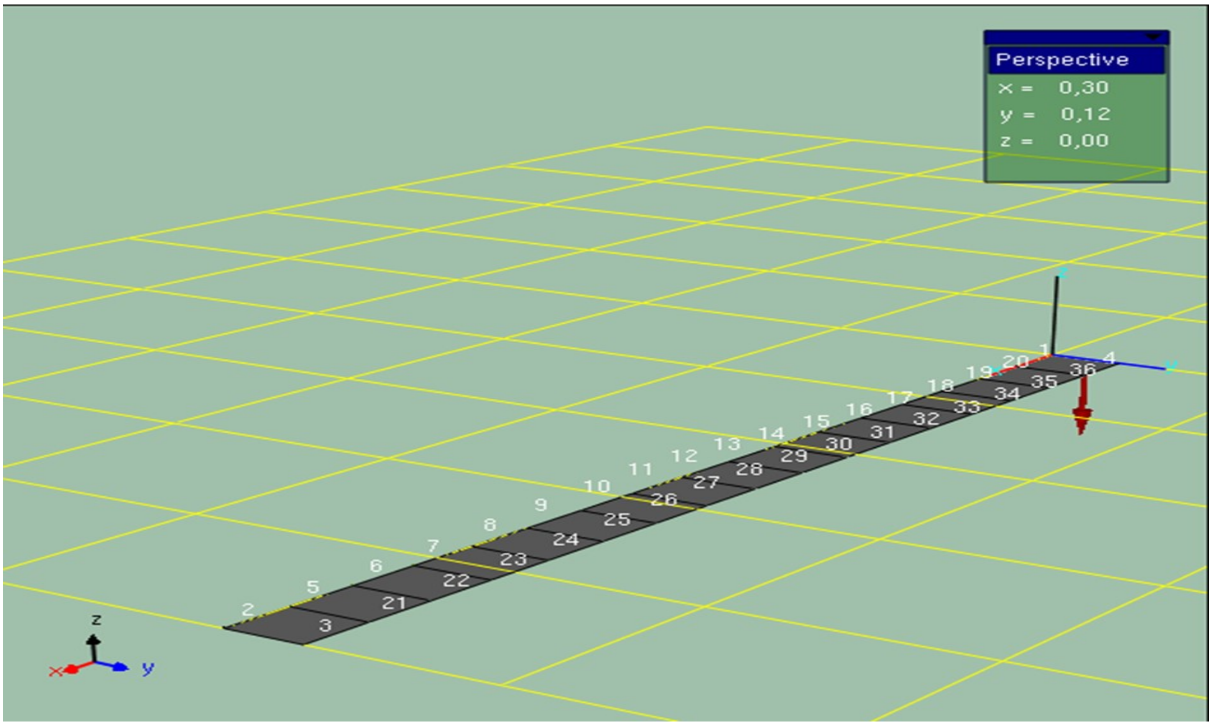

Figure 14-Designed single beam for experimental analysis

After this step 36 number of points described by program as node and point 36 selected as the reference point(red arrow on figure 14) which is accelerator connected. As an accelerator Brüel & Kjaer Delta Tron Tye 450 is selected.

To perform the experiment ,Brüel & Kjaer Type 8202 hammer(figure 15) is selected. Results are achieved by hitting all nodes with a hammer.

Figure 15-The hammer which used in experiment

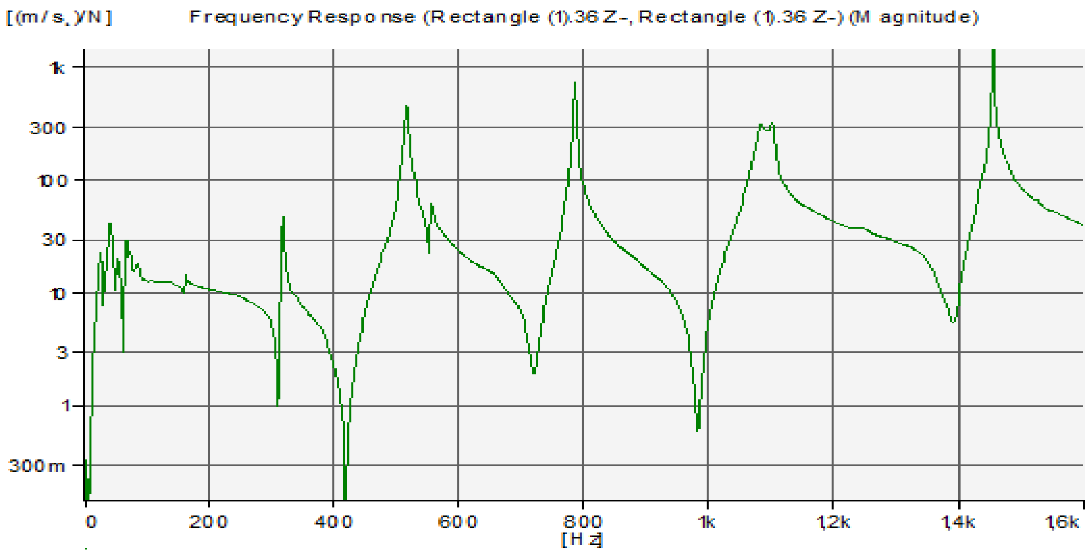

Figure 16- Frequency response function at the reference point 36

Natural frequencies and shapes were determined based on the frequency response function.

As a result frequencies(Hz) obtained by hammering can be seen on the table 2.

Table 2-Results of experiment

	Mode	Frequency (Hz)	Damping (%)	Complexity
Rigid body	1	23.97683	6.95406	0.36111
Rigid body	2	44.54107	2.37588	0.93296
1st bending	3	63.26287	1.46737	0.5994
2nd bending	4	159.43123	1.05916	0.29203
3rd bending	5	315.44848	0.30873	0.00669
1st torsion	6	514.59127	0.56001	0.05806
4th bending	7	531.24868	0.51492	0.581
5th bending	8	784.19597	0.1636	0.00772
6th bending	9	1085.9235	1.19261	0.42125
7th bending	10	1454.7722	0.18048	0.04815

5.3.Comparison of Computational and Experimental Models (Single Beam)

The comparison of computational and experimental models can be seen in table3.

Table 3-Comparisoon of frequencies(Hz)

	Mode	Frequency (Hz) Experiment	Calculated Frequency(Hz)
1st bending	1	63.26287	62.455
2nd bending	2	159.43123	172.18
3rd bending	3	315.44848	337.61
1st torsion	4	514.59127	587.90
4th bending	5	531.24868	558.24
5th bending	6	784.19597	834.22
6th bending	7	1085.9235	1165.7

5.4 FEM Modal Analysis for Whole Assembly Using Standard Model

Standard model contains two element types - beams and springs. The beam element is determined by the length, cross section and material. The beam element is used for modeling of horizontal beams, approx. 20 elements for one horizontal beam (one body).

Spring element is determined only by it’s stiffness. The spring element is used for modeling of vertical springs, one element for one spring, totally 6 spring elements, three on left and three on right side.

FEM Modal Analysis for whole assembly done by using ANSYS MECHANICAL APDL. Natural frequencies are :

Table 4-Natural frequencies found with FEM Modal analysis

no	Natural Frequencies(Hz)
1	14.658
2	24.441
3	26.099
4	28.496
5	69.950
6	72.137
7	87.315
8	117.13
9	151.60
10	184.66
11	208.49
12	237.98
13	267.64
14	267.64
15	267.64
16	294.24
17	294.24
18	294.24
19	379.13
20	390.88

First 3 modal shapes are shown below on the figures.

For the 1st modal shape, displacements on selected nodes in Y direction shown below on the figure 17.Frequency defined as 14.6575 Hz for first modal shape.

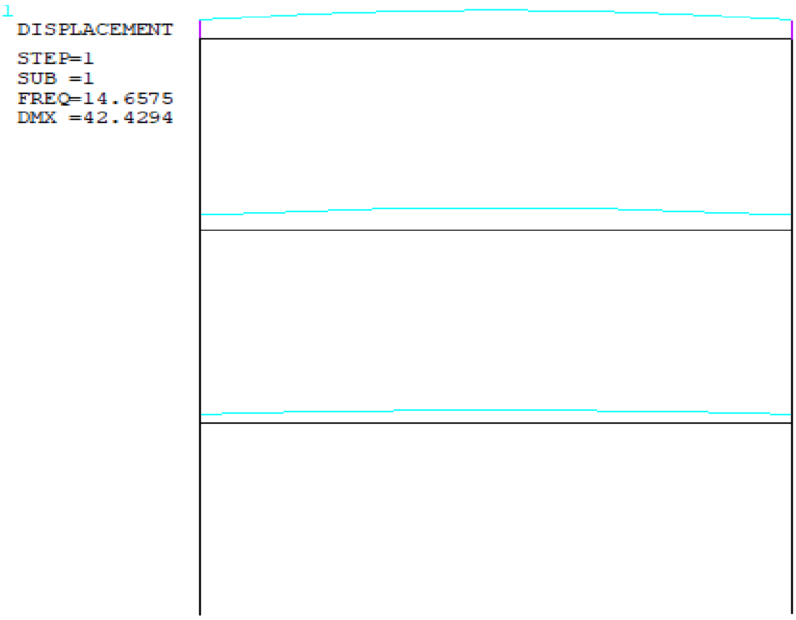

Figure 67-1st mode shape for whole assembly, f = 14.658 Hz

For the 2nd modal shape, displacements on selected nodes in Y direction shown below on the figure 18.Frequency defined as 24.4407 Hz for second modal shape.

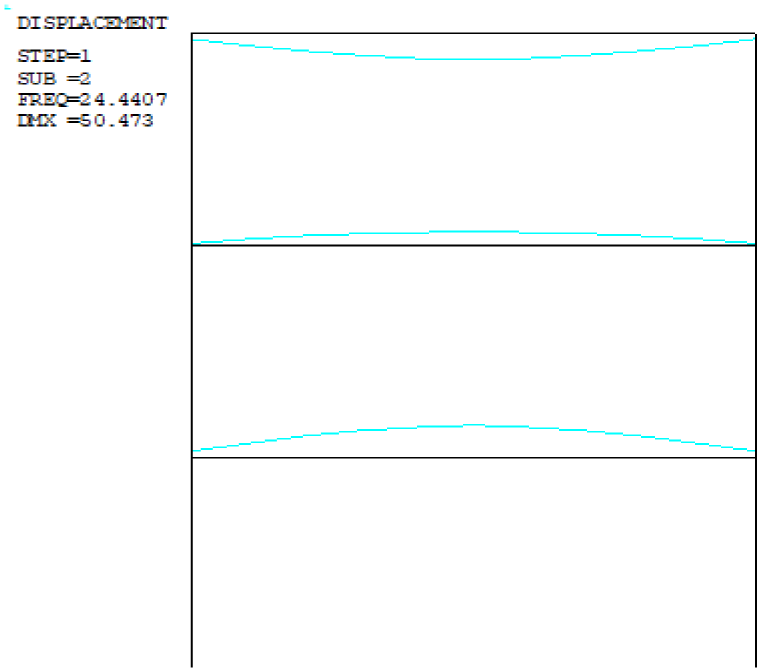

Figure 78-2nd mode shape for whole assembly, f = 24.441 Hz

For the 3rd modal shape, displacements on selected nodes in Y direction shown below on the figure 19.Frequency defined as 26.0986 Hz for third modal shape.

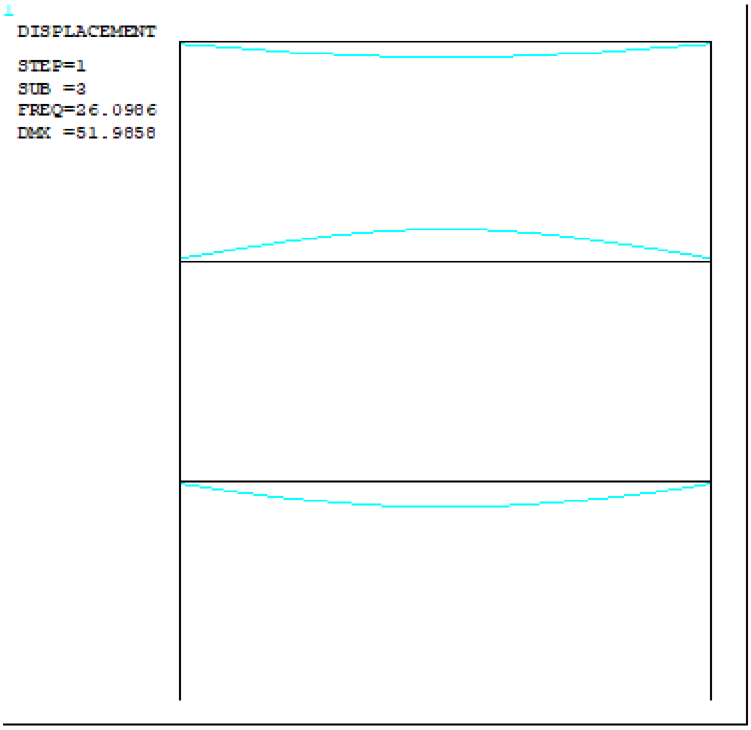

Figure 19-3rd mode shape for whole assembly, f = 26.099 Hz

5.5.FEM Modal Analysis for Whole Assembly Using Data of Substurctures

The single horizontal beams were defined as substructures. On one beam 4 nodes were selected as masters, displacement in Y axis.

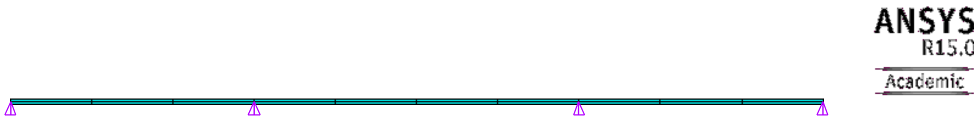

Figure 20- One beam as substructure, masters

The bottom super-element - masters are nodes 1, 5, 9, 2

Stiffness matrix is

$$\begin{bmatrix} 7.9983022 & 6.2927959 & -1.8735459 & -12.417552 \\ 1.8735459 & 6.2927959 & 7.9983022 & -12.417552 \\ -12.417552 & 7.9983022 & 27.729443 & -23.310193 \\ 7.9983022 & -12.417552 & -23.310193 & 27.729443 \end{bmatrix} \quad (19)$$

Mass matrix is

$$\begin{bmatrix} 0.34001459E-04 & 0.24051518E-05 & 0.14032663E-04 & -0.77725838E-05 \\ 0.24051518E-05 & 0.34001459E-04 & -0.77725838E-05 & 0.14032663E-04 \\ 0.14032663E-04 & -0.77725838E-05 & 0.11906296E-03 & 0.24509386E-04 \\ -0.77725838E-05 & 0.14032663E-04 & 0.24509386E-04 & 0.11906296E-03 \end{bmatrix}$$

(20)

The top super-element - masters are nodes 23, 27, 31, 24.

The middle super-element - masters are nodes 12, 16, 20, 13.

The total model contains just 9 elements - three substructures as three so called “super-elements”, and six springs and 12 DOF, the Y displacement in 12 nodes, defined as masters. (The figure of the model with super-elements looks the same, as the one of standard model, see fig 10.)

All frequencies(Hz) shown below on the table 5.

Table 5-Frequencies

	Frequency (Hz) super-elements	**Frequency (Hz) standard model**
1	14.662	14.658
2	24.471	24.441
3	26.139	26.099
4	28.534	28.496
5	70.483	69.950
6	73.744	72.137
7	88.274	87.315
8	123.80	117.13
9	167.49	151.60
10	185.07	184.66
11	209.92	208.49
12	242.21	237.98
13		267.64
14		267.64
15		267.64
16		294.24
17		294.24
18		294.24
19		379.13

Comparison of mode shapes.

Table 6-Comparison of displacements at 14.658Hz

f_1 = 14.658 Hz	
standard	**super-elements**
1. 12.677	1. 12.686
2. 12.677	2. 12.686
5. 17.714	5. 17.726
9. 17.714	9. 17.726
12. 22.843	12. 22.859
13. 22.843	13. 22.859
16. 31.920	16. 31.941
20. 31.920	20. 31.941
23. 28.484	23. 28.505
24. 28.484	24. 28.505
27. 39.804	27. 39.830
31. 39.804	31. 39.830

Table 7-Comparison of displacements at 24.441Hz

f_2 = 24.441 Hz	
standard	**super-elements**
1. 9.5760	1. 9.6059
2. 9.5760	2. 9.6059
5. 42.698	5. 42.805
9. 42.698	9. 42.805
12. 4.2617	12. 4.2750
13. 4.2617	13. 4.2750
16. 19.002	16. 19.050
20. 19.002	20. 19.050
23. -7.6794	23. -7.7033
24. -7.6794	24. -7.7033
27. -34.241	27. -34.327
31. -34.241	31. -34.327

Table 8-Comparison of displacements at 26.0998Hz

f_3 = 26.0998 Hz	
standard	**super-elements**
1. -4.1111	1. -4.1271
2. -4.1111	2. -4.1271
5. -34.530	5. -34.636
9. -34.530	9. -34.636
12. 5.1264	12. 5.1464
13. 5.1264	13. 5.1464
16. 43.058	16. 43.190
20. 43.058	20. 43.190
23. -2.2815	23. -2.2904
24. -2.2815	24. -2.2904
27. -19.163	27. -19.221
31. -19.163	31. -19.221

5.6.Experimental Modal Analysis for Whole Assembly

As a result of experimental analysis, the frequencies(Hz) of beams can be seen separately in the tables.

5.6.1.Upper Beam

Table 9-Frequencies of upper beam

	Mode	**Frequency (Hz)**
1st bending	1	33.88396
2nd bending	2	127.17901
3rd bending	3	278.35297

These frequencies also can be seen ahead in figures(21,22,23).

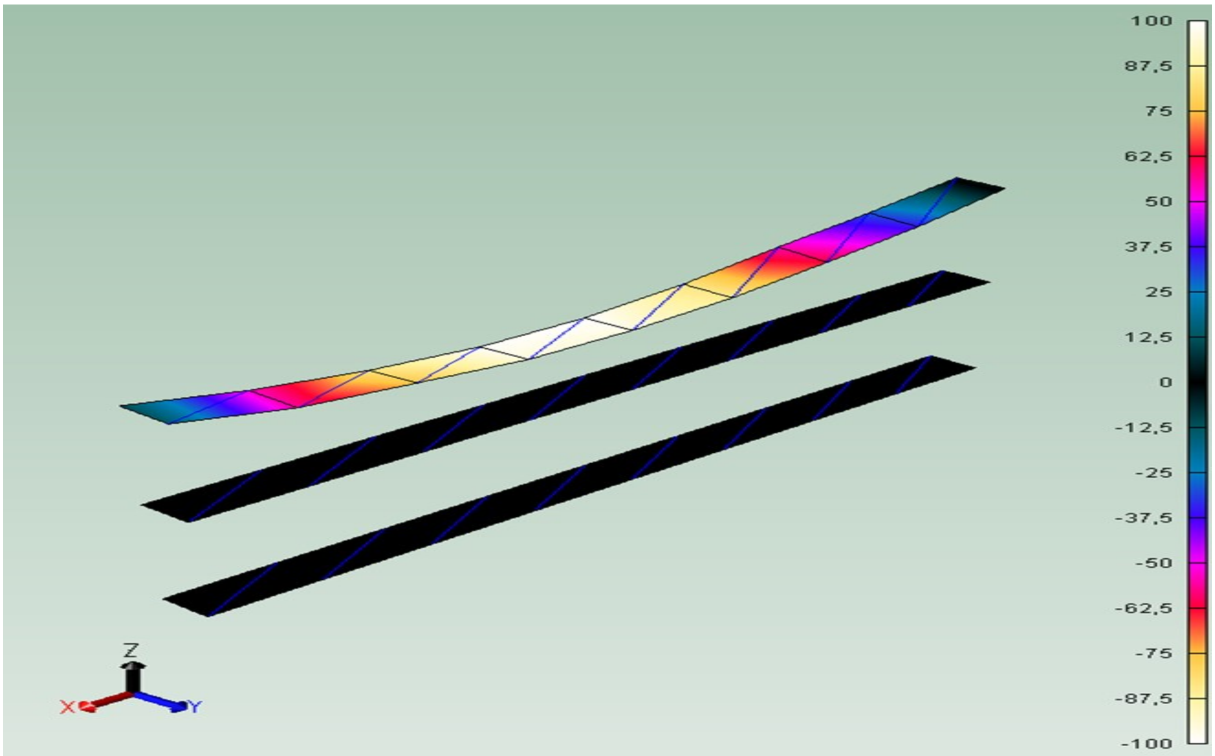

Figure 21-1st mode shape for upper beam

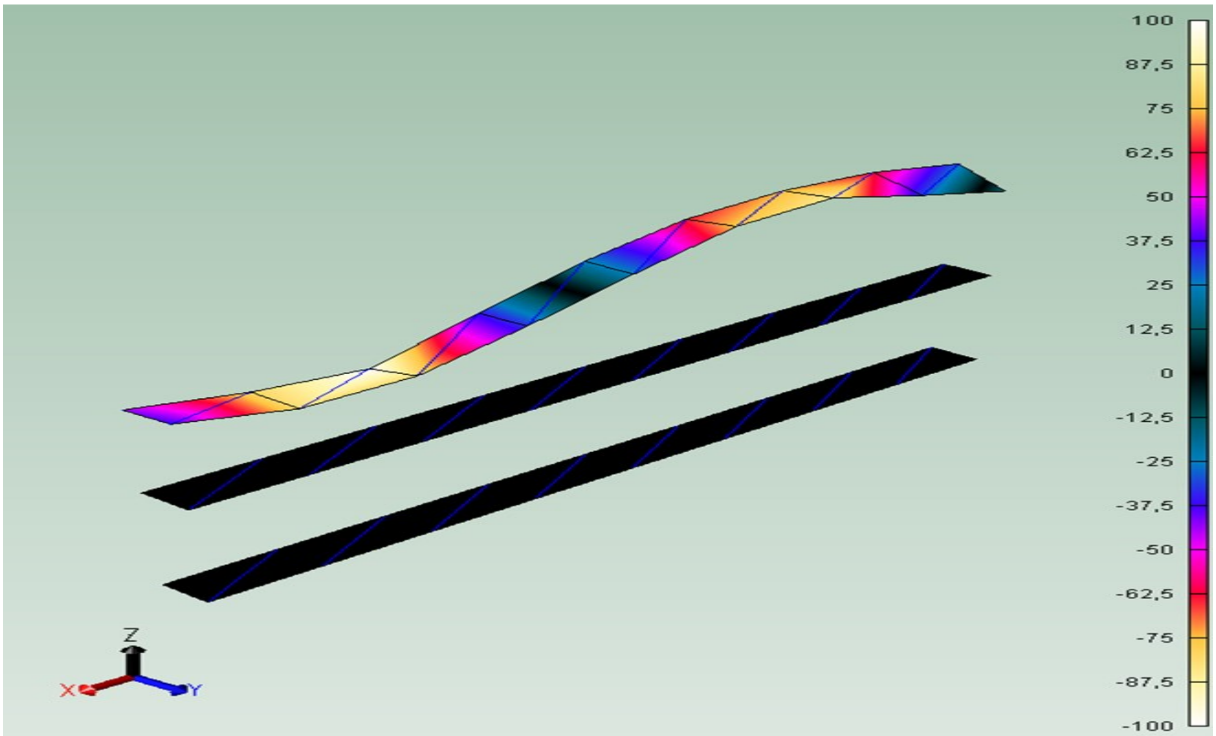

Figure 22-2nd mode shape for upper beam

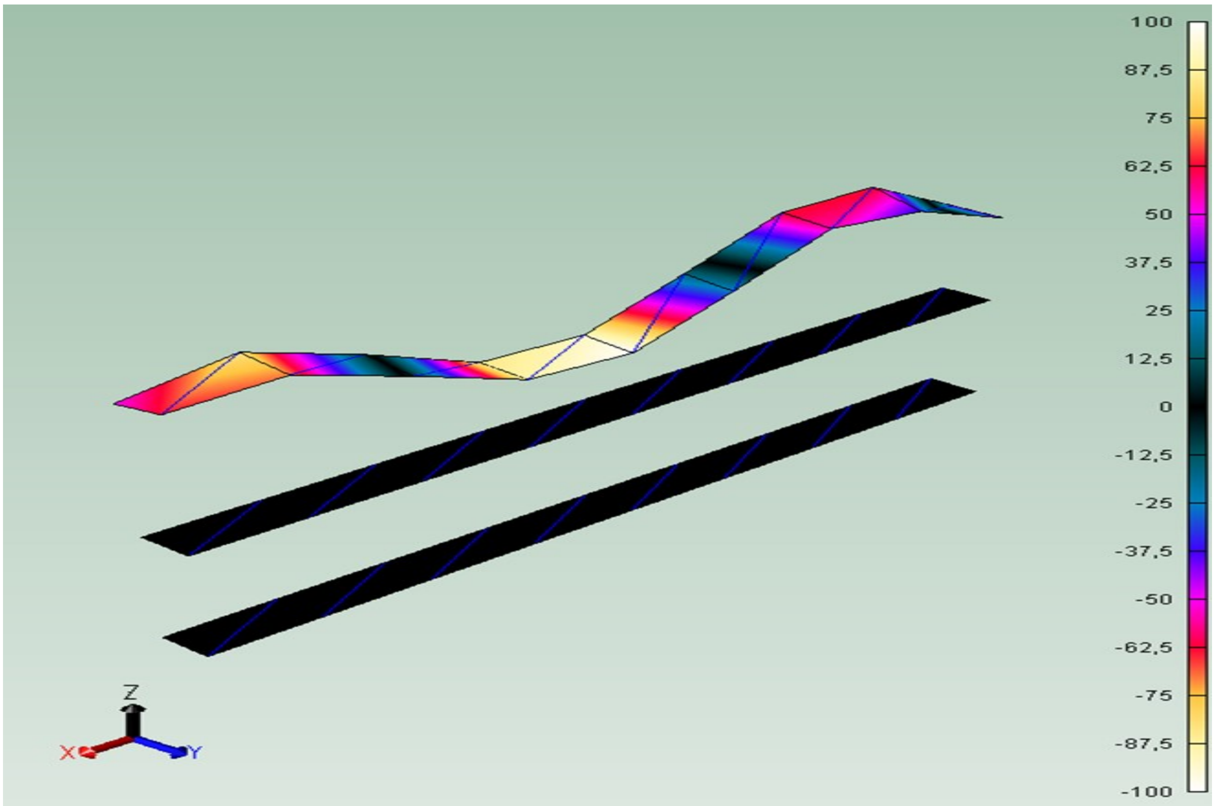

Figure 23-3rd mode shape for upper beam

5.6.2.Middle Beam

Table 10-Frequencies of middle beam

	Mode	Frequency (Hz)
1st bending	1	32.8
1st torsion	2	404,1

5.6.3.Bottom Beam

Table 11-Frequencies of bottom beam

	Mode	Frequency (Hz)
1st bending	1	31.9
2nd bending	2	144.2
1st torsion	3	402

5.7.Comparison of Computational and Experimental Results

Unfortunately the real assembly did not work properly. If the vibration was exciting on on beam (for example top), the vibration did not transmit through springs to the other beams. It is necessary to improve the physical model. For this reason the frequencies and especially mode shapes, calculated on the whole assembly (doesn't matter if standard or substructures), can not be compared with measured ones.

6. Conclusion

In this work, a dynamic analysis was performed with using sub-structuring method. Sub-structuring is a procedure that condense a group of finite elements into one super-element.

The structure which is selected for this work consists 2 kind of components which are beams and spring. Beams are linked together with springs. The system is completely immobile in place. First the system is modeled in computer environment and analysis performed in ANSYS APDL software.

The material parameters of the beams are verified by experimental measurement. Unfortunately the experiment on the whole assembly does not work properly. Subsequently the modal analysis on standard model (without substructures) and on model, containing super-elements, was performed. The results were compared with very small differences.

Finally it can be state that sub-structuring as a method gives very good results.

List of Figures

List of Tables

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